Exercise 9

Find the particular solution for each of the following initial value problems:

$$(\tan x)u' + (\sec^2 x)u = 2e^{2x}, \quad u\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{2}}$$

Solution

Observe that the left side can be written as $[(\tan x)u]'$ by the product rule.

$$\frac{d}{dx}[(\tan x)u] = 2e^{2x}$$

Now integrate both sides with respect to x.

$$(\tan x)u = e^{2x} + C$$

The general solution is thus

$$u(x) = \frac{e^{2x} + C}{\tan x}.$$

Because an initial condition is given, this constant of integration can be determined.

$$u\left(\frac{\pi}{4}\right) = \frac{e^{2\frac{\pi}{4}} + C}{\tan \frac{\pi}{4}} = \frac{e^{\frac{\pi}{2}} + C}{1} = e^{\frac{\pi}{2}} \quad \rightarrow \quad C = 0$$

Therefore,

$$u(x) = \frac{e^{2x}}{\tan x}.$$